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CURRENT DIFFUSION IN CYLINDRICAL WIRES AND FUSES DURING MICROSECOND ELECTRICAL PULSES

by

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November 1970

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ABSTRACT

Existing solutions to the diffusion equation for current in a cylindrical conductor are used to provide estimates of the non-uniformity of current density during typical wire explosions. These estimates are given in the form of inequalities which are then used to specify the conditions under which current distribution and heating effect can be considered uniform.

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I. INTRODUCTION

When a condenser bank is discharged through a circuit, part of which consists of a fine wire or fuse, more Joule heating occurs in the resistive portion than elsewhere; and, if the pulse is sufficiently short, the distribution of energy in the resistive element is not uniform owing to skin effect. Under conditions of moderate overload the resistive element melts and opens the circuit. With extreme overloads a large portion of the stored energy may be deposited in the wire and a violent explosion can result. The details of this complex process have been studied in various ways for nearly two hundred years[1] but are still far from completely understood.

One aspect that has remained obscure is the penetration into the conductor of the magnetic field (current) during the pulse. Only in recent years has the theory of transient skin effect associated with single current pulses been well understood[2] and then only for the case of a constant resistance. The exploding fuse element imposes an extremely nonlinear, variable resistance on the capacitor circuit; and, for this reason has defied detailed analysis so far.

In the present paper we make use of existing solutions to the transient skin effect problem for constant resistance elements. We provide estimates of critical time constants for impulsively heated wires undergoing nonlinear resistance changes. From these estimates we shall derive criteria for judging the uniformity of heating in a variety of exploding wire and fuse situations. Our motivation arises from an interest in the fluid dynamic vaporization processes that take place in superheated metal cylinders and in the extent to which these may be influenced by nonuniformities of the electrical heating.

The existing model for a fluid dynamic vaporization[3] supposes a spatially uniform distribution of specific internal energy at the outset, i.e., at time $t = 0$. While this is not a necessary condition and may, in fact, be relatively unimportant, it simplifies the analysis considerably, as no detailed calculation of the thermal history up to vaporization need

be made. On the other hand, should it appear that skin effect is the important cause of nonuniform early heating of the wire, then the older experiments would have to be critically reappraised and improvements made in the data reduction schemes presently in use.

II. CONSTANT RESISTANCE SOLUTIONS

In a previous study[4] we have shown that the current pulse produced by a typical condenser discharge through a fuse or exploding wire element can be divided into four parts. The first of these is a linear ramp, as in Figure 1, during which current rises like $I \sim A\,t$, where A is chosen somewhat smaller than the initial slope V_0/L. Here V_0 is the initial condenser voltage and L the total circuit inductance. This stage is followed by an interval of nearly constant current which in turn gives way to a quasi-exponential decline. If an arc forms along the wire during this vigorous damping process the final stage consists of damped oscillations. If not, the current falls to zero and under certain conditions may remain at this value. If the remaining voltage is sufficiently high damped oscillations reappear at the end of a suitable "dwell" time. The first three stages, viz., those prior to the damped oscillations, resemble an overdamped pulse of current. Since the resistance operating to produce the real pulse is variable and nonlinear, the latter part of the pulse is considerably distorted from that to be expected in the case of constant resistance. It will be assumed for this discussion that the differences can be overlooked, at least in first approximation, and use can be made of the constant resistance cases.

Haines[2] gives the solution for an overdamped circuit in his paper; and, following his method, a solution can be found for the linear ramp current. Analysis of these cases will suffice to give estimates for some critical time intervals during which skin effects are liable to be important. We shall quote the solutions in form suitable for the present use.

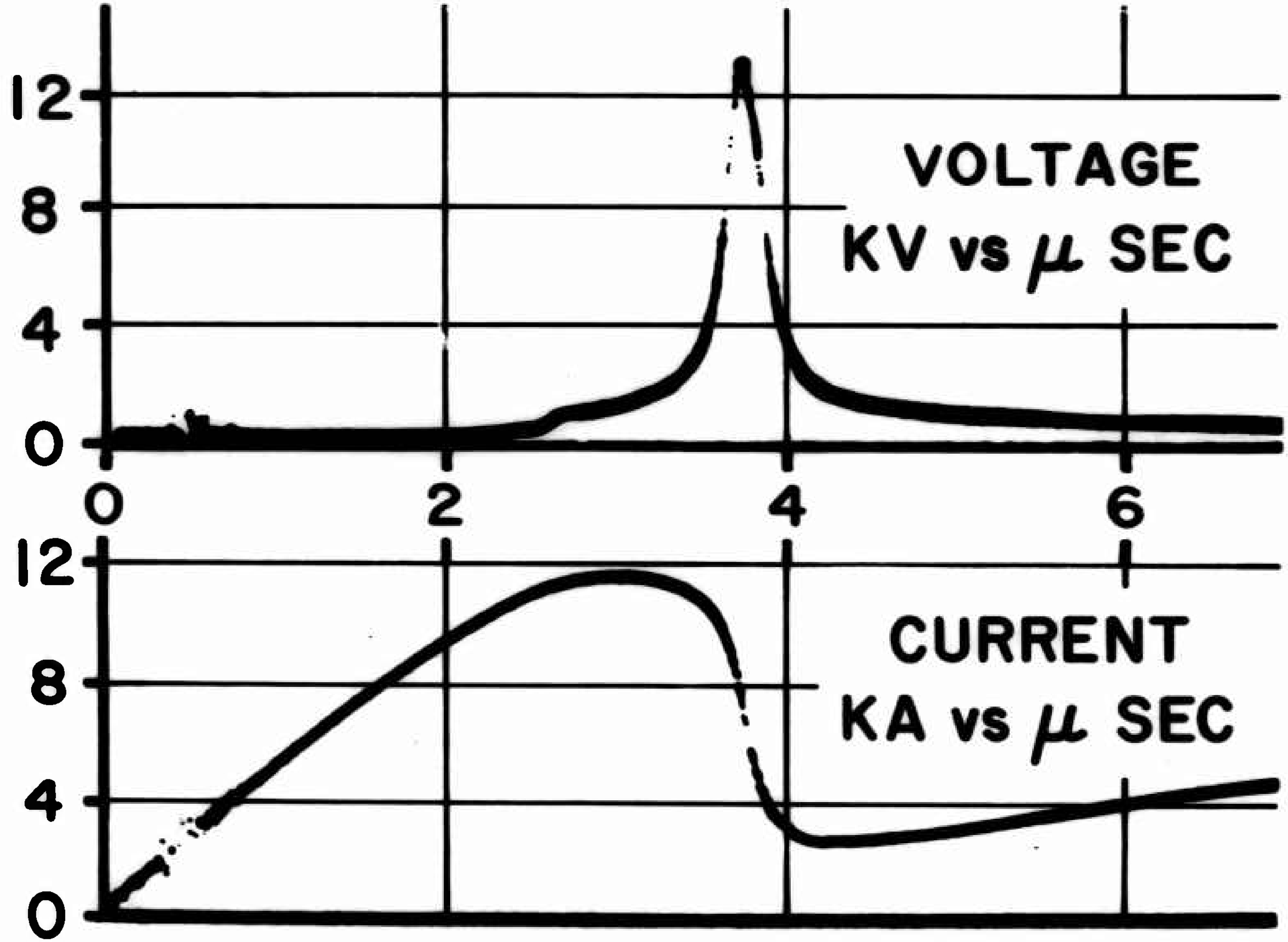

Figure 1. Representative current and voltage traces for an exploding wire. Current Ramp - 0-2.5 μsec; Constant Peak - 2.5-3.5 μsec; Vaporization Wave Decay - 3.5-4.1 μsec; Arc Breakdown - from 4.1 μsec.

A. Current Ramp Solution

For a round, cylindrical wire of radius a, permeability μ and conductivity σ, in mks units, we define $x = r/a$ and $y = t/\mu\sigma a^2$ as dimensionless variables with $\tau = \mu\sigma a^2$ for later convenience.

If we assume that at switch-on, $t = 0$, the total current increases as $I = At = A\tau y$, the current density $j(x,y)$ in the axial direction is given by[5]

$$j(x,y) = \frac{A\tau}{\pi a^2}\left[y + \frac{x^2}{4} - \frac{1}{8} - \sum_{1}^{\infty} e^{-\gamma_n^2 y}\ \frac{J_o(\gamma_n x)}{\gamma_n^2\, J_o(\gamma_n)}\right] , \quad (1)$$

where the γ_n are found from $J_1(\gamma_n) = 0$. The first few roots of the Bessel function, J_1, are $\gamma_{1,2,3} = 3.83, 7.01, 10.17$. Their squares are $\sim 15, 50, 100$.

From inspection of Eq. (1) the first critical time is that for which the largest transient term in the infinite sum drops to 1/e of its value, viz.,

$$t_1 \geq \tau/\gamma_1^2 = \mu\sigma a^2/15 \quad (1a)$$

After a few e-foldings the transient terms are negligible and the infinite sum may be disregarded. Inequality (1a) is exactly the criterion given by Haines for damping of the transient skin effect.

From the remaining terms we see that uniform current density, $A\tau y/\pi a^2$, is achieved when $y >> 1/8 \geq x^2/4 - 1/8$. A factor of 20 to satisfy the inequality would assure no more than 5% deviation in current density but 10% in power. If this requirement assures sufficient uniformity of current and energy, then

$$t_2 \geq 2.5\ \mu\sigma a^2 \quad (1b)$$

must be satisfied in addition to inequality (1a). Clearly t_2 dominates t_1 by a wide margin and sets the time interval beyond which current density will be sensibly uniform.

B. Overdamped Discharge

Let us define the impressed current to be $I = e^{-\alpha y} - e^{-\beta y}$ for $y > 0$ with $\alpha', \beta' = (R_c/2L)\,[r \mp (r^2 - 1)^{1/2}]$ where $R_c = 2\sqrt{L/C}$, $r = R/R_c$, $\alpha = \tau\alpha'$ and $\beta = \tau\beta'$. We write Haines' solution[2] in the form

$$(\pi a^2)\, j(x,y) = e^{-\alpha y}\, \frac{J_0(\sqrt{\alpha}\, x)}{2J_1(\sqrt{\alpha})/\sqrt{\alpha}} - e^{-\beta y}\, \frac{J_0(\sqrt{\beta}\, x)}{2J_1(\sqrt{\beta})/\sqrt{\beta}}$$

$$-\sum_{1}^{\infty} e^{-\gamma_n^2 y}\, \frac{J_0(\gamma_n x)}{J_0(\gamma_n)} \left[\left(\frac{\alpha}{\gamma_n^2} - 1\right)^{-1} - \left(\frac{\beta}{\gamma_n^2} - 1\right)^{-1}\right], \tag{2}$$

with $J_1(\gamma_n) = 0$ as before.

As one would expect, the damping condition for the transient terms is the same as inequality (1a); furthermore, after considering the expansions of J_0 and J_1 for small values of the argument it is evident that the coefficients of the remaining exponential terms do not approach unity unless $\sqrt{\alpha}$ and $\sqrt{\beta}$ are restricted to suitably small values. The condition on J_0 is the dominating one. In order that $0.95 \le J_0(\sqrt{\alpha}) \le 1$ we must require that $\sqrt{\alpha}x \le \sqrt{\alpha} < 0.15$. In this interval $2J_1(\sqrt{\alpha}) \sim \sqrt{\alpha}$ holds so that the ratios in Eq. (2) are unity within 2 - 3%. Thus, we may write the further requirements that

$$\alpha'\tau < \beta'\tau \le b^2 \tag{2a}$$

where $b^2 = .0225$ in the present case.

Using the definition that $\tau^* = \tau/\tau_0$ where $\tau_0 = \sqrt{LC}$, we can put inequality (2a) in the form

$$\tau^* \,[r + (r^2 - 1)^{1/2}] \le b^2 \tag{2b}$$

with $r \ge 1$ and the positive sign chosen in order to select the smallest bound on τ^*. This inequality puts a restriction on the characteristic time τ which can only be satisfied by suitable limitations on combinations of μ, σ and a.

To simplify the discussion we assume that all wires and fuses of interest do not differ magnetically from free space; thus, μ is constant and equal to $4\pi/10^7$. The interesting case of ferromagnetic conductors will be bypassed for the present. Note, however, that μ and σ enter the time constant τ in equivalent ways. As a result, regions of field intensity where μ is large will be subject to a skin effect similar to that caused by conductivity. A "permeability" skin effect may be responsible for some of the peculiar behavior of iron wires exhibited in electrical and optical data observed prior to melt (1559°C). The magnetic Curie temperature occurs at 769°C.

Table I contains values representative of some of the condenser discharge circuits used in exploding wire experiments together with the critical value $\tau_c = b^2\tau_o$ necessary to satisfy inequality (2b) and thereby assure that the overdamped discharge would take place with practically constant current distribution over the cross-section of the conductor.

Table I

L (μH)	C (μF)	R_c (ohms)	τ_o (μsec)	τ_c (nsec)
0.25	60	0.13	3.9	88
0.5	30	0.26	3.9	88
0.5	15	0.36	2.7	61
0.5	0.5	2	0.5	11
0.25	0.5	1.4	0.35	8

L = inductance, C = capacitance, $R_c = 2\sqrt{L/C}$

$\tau_o = \sqrt{LC}$, $b^2 = .0225$ and $\tau_c = b^2\tau_o$.

III. DISCUSSION

A. Current Ramp Solution

The typical pulse, shown in Figure 1, for a well-matched wire[3] has a linear ramp, a nearly constant current and a rapid decay typical of the vaporization-wave rise in resistance.

During the linear regime the wire resistance is mostly below the critical damping value although it may exceed it near the end of the ramp. In the case of wires larger than 2.5×10^{-4} m in diameter, the resistance may not exceed the critical value until late in the vaporization phase. When melting of the wire occurs, the sudden jump in resistance sometimes causes a sharp slope discontinuity in the current which then enters its nearly constant phase. If no slope discontinuity appears a relatively constant plateau is described anyway. Rapid heating occurs until the vaporization wave commences at the surface, passes inward over the wire cross section and renders it nonconducting. During this process the resistance rises steeply, and the remaining energy, stored in the magnetic field, is dumped into the diminishing cross section of the conductor.

For the study of vaporization and other fluid dynamic phenomena occurring in wires and fuses, knowledge of the thermodynamic state of the material conductor is essential. To prepare the wire in a uniform thermodynamic state via the electric pulse one must deposit the energy as uniformly as possible. Both current diffusion and vaporization phenomena tend to defeat this objective; the one because of nonuniform current distribution, the other because interior parts of the conductor are heated longer than those annuli vaporized from the outside. For the present we attempt to specify the conditions under which the electrical heating can be uniform within tolerable limits.

On the current ramp inequality (1b) sets the interval, i.e., $\tau_2 \geq 2.5\ \tau$. Whatever the value of $\tau = \mu\sigma a^2$, the transient terms will have long been damped out by the time t_2 has elapsed. One would like

inequality $\tau_2 \ll \tau_0$ to hold in order that uniform heating prevail over most of the first quarter cycle, a time of order $\sim\sqrt{LC}$. This requirement may be too severe in some practical situations for the reason that while current is rising linearly with time the Joule heat deposited increases initially like current cubed, and more rapidly thereafter[8]. If the material density, specific heat and temperature coefficient of resistivity remain approximately constant during the linear rise of current prior to melting of the wire, we have shown that conservation of energy implies conductivity to decrease from the room temperature σ_0 as $\sigma = \sigma_0 e^{-Bt^3}$, where B is a constant. The first e-folding time is $B^{-1/3}$ μsec. For some typical experiments in a slow circuit of Table 1 with τ_o = 3.9 μsec and a current rise A = 4.8×10^9 amp/sec, B = 0.1 μsec^{-3} for 10 mil Cu wire. In a faster circuit, τ_o = 0.5 μsec, A = 4×10^{10} amp/sec and B = 70 μsec^{-3} for 5 mil Cu wire. Clearly, τ decreases according to the same law as σ under the conditions stated above.

Table II shows the critical time, τ, at room and melt temperatures for cylindrical wires of several different metals and diameters. When the wire has completely melted, the conductivity has dropped by about an order of magnitude and the critical time likewise. Notice the general result that heating through melt greatly diminishes both the conductivity and the skin effect in a conductor.

Table II

Element	δ	(σ_R/σ_M)	τ_R (nsec)	τ_M	$\tau_R/15$
Cu	1	12.	290	25	20
	2		1160	100	80
	5		7200	600	480
Al	1	8.6	180	20	12

Table II (Continued)

Element	δ	(σ_R/σ_M)	τ_R (nsec)	τ_M	$\tau_R/15$
	2		720	80	50
Pb	1	4.3	20	5	1
	3		180	45	12
Fe	1	13.	50	4	3
	5		1250	100	80

σ = conductivity; subscripts R and M refer to room and melt (complete) temperatures respectively; $\tau = \mu\sigma a^2$, δ is the wire diameter in mils divided by 5; i.e., the .005" wire is taken as standard. All figures are approximate.

Comparison of 2.5 τ_R and 2.5 τ_M with the τ_0 values of Table I shows that above the melting temperature uniform current distribution is quickly established for all wires with $\delta = 1$. In the faster circuits nonuniformities owing to skin effect may be expected for δ's of 2 and 5. Some care must be used to choose wires small enough.

Below melting and at room temperature, only the $\delta = 1$ wires heated in the slower circuits will satisfy the condition for uniformity imposed by inequality (1b). In particular, according to this criterion the larger copper wires must exhibit substantial nonuniformities of current density. The approximate magnitude of the effect may be seen by referring to Eq. (1) and the arguments leading to Eq. (1b). Supposing, for the sake of illustration, that $\langle\tau\rangle = (\tau_R + \tau_M)/2$ is a reasonable value to characterize the wire behavior between room temperature and melt, then for $\delta = 2$ and copper wire, $\langle\tau\rangle = 630$ nsec and until y = 2 the current density will vary by 12% or more from periphery to center. The variation in heating, i.e., in specific energy, will be more than 35%. This estimate only makes sense for the slower circuits, $\tau_0 \sim 3$-4 μsec, and is considerably mitigated by the fact that most of the heating occurs near melt and beyond. For the faster circuit the explosion

occurs by $y \sim 0.5$-1.0 and the nonuniformity in current density may exceed 25% over most of the pulse interval. Concomitant variations in heating and temperature may exceed a factor of 2.

For the largest Cu wire, $\delta = 5$, the critical time τ is of the same order as the pulse time in the slower circuits, i.e., the pulse occurs in dimensionless times $y \leq 1$. The estimates just given, viz. 25% and 2:1, will still apply. If one were to explode the $\delta = 5$ wire in the faster circuits much larger nonuniformities in current density and specific energy would be expected as the transient terms could no longer be ignored.

B. Overdamped Solution

When the current begins to fall appreciably below the linear ramp, the broad peak of the curve is at hand. This is the interval, before onset of the vaporization wave, during which current is nearly constant. We approximate this region by use of the solution for the overdamped discharge given in Eq. (2). Since the maximum of current occurs in this interval, both the exponential terms must be retained, although beyond the peak the first term would suffice, for it dominates the decay portion of the pulse. As melt frequently occurs near current peak, and as the wire resistance, R, is still comparatively low at or near the time of melt, and as wire resistance typically does not exceed a few ohms until the later passage of the vaporization wave, we assume that $r \approx 1$ is a reasonable assumption for inequality (2b) which then allows the maximum value for τ^*. The corresponding limits on characteristic time τ_c are tabulated in the last column of Table I. Comparison with τ_M in Table II shows that uniform current density within about 5% can be expected around current maximum in the slower circuits for all wires except those of copper with $\delta = 5$. In the faster circuits, with wires of $\delta \geq 1$, considerably larger nonuniformities in current distribution might be expected. Thus, there are good reasons for choosing finer wires, $\delta < 1$, for use as the frequency of the circuit increases. One of these, of course, has to do with the smaller energy storage usual in

a fast circuit with small capacity, but skin effect considerations may offer the more important reason for choosing smaller wires.

The case for uniformity of current density during current maximum can be further argued on the grounds that if $j(x,y)$ is uniform on and near the end of the current ramp, there is little change in current during the interval around the peak, $\tau \le \tau_M$ ought to hold and any nonuniformities ought to be confined to the peripheral layers. Because of the nonlinear behavior of R, the overdamped solution is at best only approximate; and, as stated above, errs toward being too strict since the bound (2b) is based on the larger root, β', which in turn affects mainly the rising portion and peak of the discharge curve. Once over the peak the smaller root would apply and the bound increases rapidly with r, thus relaxing the restriction on τ.

C. Vaporization Wave

After current peak the anomalous decay of current is directed and dominated by resistance changes occurring because of the rapid decrease in conducting cross section of the wire. The vaporization expansion wave proceeds inward from the periphery of the wire and transforms the melted, superheated metal from liquid to a nonconducting vapor[7]. In this regime of specific energies which exceed that of the normal boiling point, the resistivity is several times higher than that at the melting point, for example, 2-4 times in the case of copper. Therefore, τ is several times smaller than τ_M given in Table II and for wires of $\delta \le 2$ ought to be less than 50 nsec. The total time of wave passage, τ_w , may be as large as 500 nsec and $\tau_w/\tau > 10$ seems to hold for all our examples.

We have then an indication that current density during wave passage ought to be reasonably uniform. Admittedly this entire regime is characterized by lines of current flow crowding from outer annuli of the wire, where vaporization has rendered the fluid nonconducting, into the interior which is inertially confined, still conducting and rapidly being heated. For the larger wires $\delta \ge 2$, nonuniformities in current

density and heating from this cause are to be expected in addition to those which arise simply because the core conducts longer than the periphery[7].

IV. SUMMARY AND CONCLUSIONS

In the preceding paragraphs we have analyzed the current pulse carried by a wire or fuse during an explosive disintegration. The real pulse is then approximated in sections, e.g., ramp, constant current and vaporization wave decay. Then appeal is made to the existing skin-effect solutions for constant resistance circuits carrying linear ramp or overdamped-discharge currents.

The results are presented in the form of several inequalities on the characteristic time $\tau = \mu\,\sigma\,a^2$. Transient skin effects damp out in times of the order $\tau/15$; while uniform current within 5% on the ramp can only be expected after times like 2.5 τ. To assure negligible skin effects around the current maximum, where the heating effect becomes important, the characteristic time ought to be no more than about 2% of the circuit pulse time measured as the quantity $\sqrt{LC}$.

In a real event the characteristic time steadily decreases as the conductivity does. By the time melt is completed, conductivity has fallen by about an order of magnitude and skin effects are correspondingly diminished. If the principal deposits of energy can be arranged to occur after the wire or fuse is melted, the distribution of energy ought to be uniform in all but the largest wires considered.

Because of the complexity of the wire explosion and of the skin-effect even for conditions of constant resistance, each experimental situation must be analyzed carefully with the limiting inequalities in mind. In particular, experiments requiring large wires, e.g., X-ray studies of striations and other density effects which may be difficult to resolve, are liable to require conductors so large that uniformity of heating by the electrical pulse can no longer be guaran i.

It is shown that simple appeal to steady-state, c.w. skin depth formulae is not sufficient to answer questions concerning current diffusion and uniformity of heating in transient, pulse experiments.

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13. ABSTRACT

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